

Combining $K^0 - \bar{K}^0$ mixing and $D^0 - \bar{D}^0$ mixing to constrain the flavor structure of new physics

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New physics at high energy scale often contributes to $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings in an approximately $SU(2)_L$ invariant way. In such a case, the combination of measurements in these two systems is particularly powerful. The resulting constraints can be expressed in terms of misalignments and flavor splittings.

Introduction. Measurements of flavor changing neutral current processes put strong constraints on new physics at the TeV scale and provide a crucial guide for model building. In particular, measurements of the mass splitting and CP violation in the neutral K system [1],

$$\begin{aligned}\Delta m_K/m_K &= (7.01 \pm 0.01) \times 10^{-15}, \\ \epsilon_K &= (2.23 \pm 0.01) \times 10^{-3},\end{aligned}\quad (1)$$

require a highly non-generic flavor structure to any such theory. Recently, huge progress has been made in measurements of the mass splitting and in the search for CP violation in the neutral D system [2]:

$$\begin{aligned}\Delta m_D/m_D &= (8.6 \pm 2.1) \times 10^{-15}, \\ A_\Gamma &= (1.2 \pm 2.5) \times 10^{-3}.\end{aligned}\quad (2)$$

These measurements are particularly useful in constraining models where the main flavor changing effects occur in the up sector [3].

By ‘non-generic flavor structure’ we mean either alignment or degeneracies or both. Each of the set of constraints (1) and (2) can be satisfied by aligning the new physics contributions with specific directions in flavor space. However, contributions that involve only quark doublets cannot be simultaneously aligned in both the down and the up sectors. Thus, the combination of the measurements related to $K^0 - \bar{K}^0$ mixing (1) and to $D^0 - \bar{D}^0$ mixing (2) leads to unavoidable bounds on new physics degeneracies.

In this work, we develop the formalism that is necessary to obtain these unavoidable bounds, explain the qualitative implications and derive the actual quantitative constraints from the present experimental bounds.

Theoretical and experimental background. The effects of new physics at a high scale $\Lambda_{\text{NP}} \gg m_W$ on low energy phenomena can be expressed in terms of an effective Hamiltonian, composed of Standard Model (SM) fields and obeying the SM symmetries. In particular, four-quark operators contribute to $\Delta S = 2$ and $\Delta C = 2$ processes. We are interested in the operators that involve

only quark doublets:

$$\frac{1}{\Lambda_{\text{NP}}^2} [z_1^K (\bar{d}_L \gamma_\mu s_L) (\bar{d}_L \gamma^\mu s_L) + z_1^D (\bar{u}_L \gamma_\mu c_L) (\bar{u}_L \gamma^\mu c_L)]. \quad (3)$$

We constrain new physics by requiring that contributions of the form (3) do not exceed the experimental value of Δm_K and the one-sigma upper bounds on Δm_D and on CP violation in $D^0 - \bar{D}^0$ mixing. As concerns ϵ_K , since the SM contribution has only little uncertainties and should be taken into account, we require that the new physics is smaller than 0.6 times the experimental bound [4]. We update the calculations of Ref. [5] (the details are presented in [3]) and obtain the following upper bounds on $|z_1^K|$ and $|z_1^D|$:

$$\begin{aligned}|z_1^K| &\leq z_{\text{exp}}^K = 8.8 \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2, \\ |z_1^D| &\leq z_{\text{exp}}^D = 5.9 \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2,\end{aligned}\quad (4)$$

and on $\text{Im}(z_1^K)$ and $\text{Im}(z_1^D)$:

$$\begin{aligned}\text{Im}(z_1^K) &\leq z_{\text{exp}}^{IK} = 3.3 \times 10^{-9} \left(\frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2, \\ \text{Im}(z_1^D) &\leq z_{\text{exp}}^{ID} = 1.0 \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2.\end{aligned}\quad (5)$$

When effects of $SU(2)_L$ breaking are small, the terms that lead to z_1^K and z_1^D have the form

$$\frac{1}{\Lambda_{\text{NP}}^2} (\bar{Q}_{Li}(X_Q)_{ij} \gamma_\mu Q_{Lj}) (\bar{Q}_{Li}(X_Q)_{ij} \gamma^\mu Q_{Lj}), \quad (6)$$

where X_Q is an hermitian matrix. The matrix X_Q provides a source of flavor violation beyond the Yukawa matrices of the SM, Y_d and Y_u :

$$\bar{Q}_{Li}(Y_d)_{ij} d_j \phi_d + \bar{Q}_{Li}(Y_u)_{ij} u_j \phi_u. \quad (7)$$

Here $\phi_{d,u}$ are Higgs doublets of opposite hypercharges. (Within the SM, $\phi_u = \tau_2 \phi_d^\dagger$.) Without loss of generality, we can choose to work in a basis where

$$Y_d = \lambda_d, \quad Y_u = V^\dagger \lambda_u, \quad X_Q = V_d^\dagger \lambda_Q V_d, \quad (8)$$

where $\lambda_{d,u,Q}$ are diagonal real matrices, V is the CKM matrix, and V_d is a unitary matrix which parameterizes the misalignment of the operator (6) with the down mass basis. If we minimize the number of phases in V (namely V depends on three mixing angles and a single phase), then V_d depends on six parameters: three real mixing angles and three phases.

Alternatively, we can choose to work in a basis where

$$Y_d = V\lambda_d, \quad Y_u = \lambda_u, \quad X_Q = V_u^\dagger \lambda_Q V_u. \quad (9)$$

Here V_u is a unitary matrix which parameterizes the misalignment of the operator (6) with the up mass basis: $V_u = V_d V^\dagger$.

The X_Q -related contributions to the operators (3) have the form

$$\begin{aligned} z_1^K &= (z_{12}^d)^2 \equiv [(V_d^\dagger \lambda_Q V_d)_{12}]^2, \\ z_1^D &= (z_{12}^u)^2 \equiv [(V_u^\dagger \lambda_Q V_u)_{12}]^2. \end{aligned} \quad (10)$$

Two generations, no CP violation. The experimental constraints that are most relevant to our study are related to $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixing, which involve only the first two generation quarks. When studying new physics effects, ignoring the third generation is often a good approximation to the physics at hand. Indeed, even when the third generation does play a role, our two generation analysis is applicable as long as there are no strong cancellations with contributions related to the third generation.

In a two generation framework, V depends on a single mixing angle (the Cabibbo angle θ_c), while V_d depends on a single angle and a single phase. To understand various aspects of our analysis, it is useful, however, to provisionally set the phase to zero, and study only CP conserving (CPC) observables. We thus have

$$\begin{aligned} \lambda_Q &= \text{diag}(\lambda_1, \lambda_2), \\ V &= \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}, \\ V_d &= \begin{pmatrix} \cos \theta_d & \sin \theta_d \\ -\sin \theta_d & \cos \theta_d \end{pmatrix}. \end{aligned} \quad (11)$$

It is convenient to define

$$\lambda_{12} = \frac{1}{2}(\lambda_1 + \lambda_2), \quad \delta_{12} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}, \quad \Lambda_{12} = \delta_{12} \lambda_{12}. \quad (12)$$

Here λ_{12} parameterizes the overall, flavor-diagonal suppression of X_Q (in particular, loop factors), while δ_{12} parameterizes suppression that is coming from approximate degeneracy between the eigenvalues of X_Q . Our analysis will yield upper bounds on Λ_{12} , but with reasonable assumptions about λ_{12} they can be translated into upper bounds on the flavor degeneracy factor δ_{12} .

It is straightforward to solve for z_1^K and z_1^D :

$$\begin{aligned} z_1^K &= \Lambda_{12}^2 \sin^2 2\theta_d, \\ z_1^D &= \Lambda_{12}^2 \sin^2 2(\theta_d - \theta_c). \end{aligned} \quad (13)$$

We learn that the new physics contribution to Δm_K can be set to zero by alignment in the down sector, $\theta_d = 0$, while the contribution to Δm_D can be set to zero by alignment in the up sector, $\theta_d = \theta_c$. However, one cannot set the contributions to both Δm_K and Δm_D to zero by any choice of θ_d . Thus, the combination of Δm_K and Δm_D provides an unavoidable bound on Λ_{12} . Defining

$$r_{KD} \equiv \sqrt{z_{\text{exp}}^K / z_{\text{exp}}^D}, \quad (14)$$

the weakest bound on Λ_{12} corresponds to

$$\tan 2\theta_d = \frac{r_{KD} \sin 2\theta_c}{1 + r_{KD} \cos 2\theta_c}, \quad (15)$$

and is given by

$$\Lambda_{12}^2 \leq \frac{z_{\text{exp}}^K + z_{\text{exp}}^D + 2\sqrt{z_{\text{exp}}^K z_{\text{exp}}^D} \cos 2\theta_c}{\sin^2 2\theta_c}. \quad (16)$$

Using Eq. (4), we obtain that the weakest bound occurs at $\sin 2\theta_d \approx 0.25$, and is given by

$$\Lambda_{12} \leq 3.8 \times 10^{-3} \left(\frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right). \quad (17)$$

We learn that for $\Lambda_{\text{NP}} \leq 1 \text{ TeV}$, the flavor-diagonal and flavor-degeneracy factors should provide a suppression at least as strong as $\mathcal{O}(0.004)$. To appreciate the significance of this result, the reader should bear in mind that each of the Δm_K and Δm_D bounds can be satisfied with appropriate alignment and neither flavor-diagonal suppression nor flavor-degeneracy. For tree level contributions with couplings of order one ($\lambda_{12} \sim 1$), the flavor degeneracy should be stronger than 0.004. For loop suppression, say $\lambda_{12} \sim \alpha_2$, the degeneracy should be stronger than 0.1.

Two generations, CP violation. To incorporate CP violation (CPV), one has to employ an appropriate formalism. We use the fact that the matrices z^d and z^u defined in Eq. (10) are hermitian. We use properties of hermitian matrices to parametrize z^d as follows:

$$\begin{aligned} z^d &= V_d^\dagger \lambda_Q V_d \\ &= \lambda_{12} \left(\mathbb{I} + \delta_{12} V_d^\dagger \sigma_3 V_d \right) \\ &\equiv \lambda_{12} \left(\mathbb{I} + \delta_{12} \hat{v} \cdot \vec{\sigma} \right), \end{aligned} \quad (18)$$

where σ_i are the Pauli matrices, the \hat{v}_i are real with $|\hat{v}| = 1$. The matrix z^u is related to z^d by a unitary transformation involving the Cabibbo matrix. It follows that it can be written as

$$z^u = \lambda_{12} \left(\mathbb{I} + \delta_{12} \hat{v}' \cdot \vec{\sigma} \right), \quad (19)$$

where

$$\hat{v}' = \begin{pmatrix} \cos 2\theta_c & 0 & -\sin 2\theta_c \\ 0 & 1 & 0 \\ \sin 2\theta_c & 0 & \cos 2\theta_c \end{pmatrix} \hat{v}. \quad (20)$$

Our formalism is motivated by the fact that it puts all CPV in \hat{v}_2 . The \hat{v}_2 parameter is the projection of X_Q onto the direction perpendicular to the 1–3 plane where, without loss of generality, $Y_d Y_d^\dagger$ and $Y_u Y_u^\dagger$ reside. This can be clearly seen from the expression for the Jarlskog invariant for our framework:

$$J = \text{Tr} \left\{ X \left[Y_d Y_d^\dagger, Y_u Y_u^\dagger \right] \right\} \quad (21)$$

$$= i(y_s^2 - y_d^2)(y_c^2 - y_u^2) \Lambda_{12} \sin 2\theta_c \hat{v}_2.$$

Using this parametrization, we obtain

$$z_1^K = \Lambda_{12}^2 (\hat{v}_1 - i\hat{v}_2)^2, \quad (22)$$

$$z_1^D = \Lambda_{12}^2 (\cos 2\theta_c \hat{v}_1 - \sin 2\theta_c \hat{v}_3 - i\hat{v}_2)^2. \quad (23)$$

Note that, among the three \hat{v}_i , there are only two independent parameters. We thus study the constraints as a function of

$$\sin \gamma \equiv \hat{v}_2 \in [0, 1], \quad (24)$$

$$\sin \alpha \equiv \frac{\hat{v}_1}{\sqrt{\hat{v}_1^2 + \hat{v}_3^2}} \in [-1, 1].$$

In terms of α and γ , we obtain

$$|z_1^K| = \Lambda_{12}^2 [\cos^2 \gamma \sin^2 \alpha + \sin^2 \gamma], \quad (25)$$

$$|z_1^D| = \Lambda_{12}^2 [\cos^2 \gamma \sin^2 (\alpha - 2\theta_c) + \sin^2 \gamma],$$

$$\text{Im}(z_1^K) = -\Lambda_{12}^2 \sin \alpha \sin 2\gamma,$$

$$\text{Im}(z_1^D) = -\Lambda_{12}^2 \sin (\alpha - 2\theta_c) \sin 2\gamma.$$

As a first check of our results, note that when we take $\gamma = 0$, we reproduce Eq. (13). (The identification of α with $2\theta_d$ is correct only in the CPC case.) The bound (17) remains the weakest bound on the flavor degeneracy. In the presence of a CPV phase in V_d , the bound becomes stronger. The weakest Λ_{12} -bound as a function of $\sin \gamma$ is presented in Fig. 1.

At $0.03 \lesssim |\sin \gamma| \lesssim 0.98$, the constraints from the CPV observables are dominant, and the combination of z_{exp}^{IK} and z_{exp}^{ID} is responsible for the unavoidable bound on Λ_{12} . Defining

$$r_{KD}^I \equiv z_{\text{exp}}^{IK} / z_{\text{exp}}^{ID}, \quad (26)$$

the weakest bound on Λ_{12} corresponds to

$$\tan \alpha = \frac{r_{KD}^I \sin 2\theta_c}{1 + r_{KD}^I \cos 2\theta_c}, \quad (27)$$

and is given by

$$\Lambda_{12}^2 \leq \frac{z_{\text{exp}}^{ID}}{\sin 2\theta_c \sin 2\gamma} \sqrt{1 + r_{KD}^{I2} + 2r_{KD}^I \cos 2\theta_c}. \quad (28)$$

Using Eq. (5), we find that the weakest bound occurs at $\sin \alpha \approx 0.014$ and it is given by

$$\Lambda_{12} \leq \frac{4.8 \times 10^{-4}}{\sqrt{\sin 2\gamma}} \left(\frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right). \quad (29)$$

Eq. (29) explains the $\sin \gamma$ dependence of the curve in Fig. 1 in the relevant range.

Comparison with Eq. (17) reveals the power of the upper bound on CPV in $D^0 - \overline{D}^0$ mixing in constraining the flavor structure of new physics. For maximal phases ($\sin 2\gamma = 1$), it implies degeneracy stronger by a factor of 8 compared to the bound from CPC observables. For $\Lambda_{\text{NP}} \leq 1 \text{ TeV}$ and large phases, the flavor-diagonal and flavor-degeneracy factors should provide a suppression stronger than $\mathcal{O}(10^{-3})$. With loop suppression of order $\lambda_{12} \sim \alpha_2$, the degeneracy should be stronger than 0.02.

Supersymmetry. An explicit example of the constraints on new physics parameters obtained by combining measurements of $K^0 - \overline{K}^0$ mixing and of $D^0 - \overline{D}^0$ mixing is provided by supersymmetry. Any supersymmetric model generates the operator (6) via box diagrams with intermediate gluinos and squark-doublets. The various factors that enter z_1^K and z_1^D can be identified as follows:

$$\Lambda_{\text{NP}} = \tilde{m}_Q \equiv (m_{\tilde{Q}_1} + m_{\tilde{Q}_2})/2,$$

$$\lambda_{12}^2 = \frac{\alpha_s^2}{54} g(m_{\tilde{g}}^2 / \tilde{m}_Q^2),$$

$$\delta_{12} = (m_{\tilde{Q}_2} - m_{\tilde{Q}_1}) / (m_{\tilde{Q}_1} + m_{\tilde{Q}_2}), \quad (30)$$

where $m_{\tilde{Q}_i}$ is the squark-doublet mass, $m_{\tilde{g}}$ is the gluino mass, and $g(m_{\tilde{g}}^2 / \tilde{m}_Q^2)$ is a known function (see *e.g.* [6]) with, for example $g(1) = 1$. Taking $\tilde{m}_Q \leq 1 \text{ TeV}$, and $m_{\tilde{g}} \approx \tilde{m}_Q$ (which gives $\lambda_{12} \approx 0.014$), leads to

$$\frac{m_{\tilde{Q}_2} - m_{\tilde{Q}_1}}{m_{\tilde{Q}_1} + m_{\tilde{Q}_2}} \leq \begin{cases} 0.034 & \text{maximal phases} \\ 0.27 & \text{vanishing phases} \end{cases} \quad (31)$$

We conclude that if squarks and gluinos are lighter than TeV, then the first two squark doublets should be degenerate to, at least, order ten percent. (Previous studies of

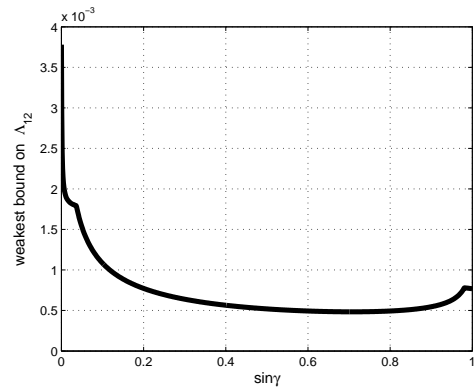


FIG. 1: The weakest Λ_{12} -bound as function of $\sin \gamma$.

the $\Delta m_K - \Delta m_D$ constraint on the degeneracy between the squark doublets can be found in Refs. [6, 7].)

Warped extra dimension. Another explicit example of the constraints on new physics parameters is provided by the Randall-Sundrum (RS) type I framework [8]. Allowing the SM fields to propagate in the bulk provides a solution to the flavor puzzle [9, 10] and also partial protection against flavor violation which is being induced, at tree level, by a Kaluza-Klein (KK) gluon exchange process [11]. In models where each SM doublet corresponds to a zero mode of a single 5d doublet, operators of the form of (6) arise. Working in a two generation framework and to leading order, we find [11, 12]

$$X_Q = \text{diag}(f_{Q^1}^2, f_{Q^2}^2), \quad \Lambda_{\text{NP}}^2 \approx \frac{3m_{\text{KK}}^2}{2\pi\alpha_s\xi}, \quad (32)$$

where $\xi \sim \ln(M_{\text{Pl}}/\text{TeV}) \sim 35$, and f_{Q^i} corresponds to the value of the i th doublet field on the IR brane. In this framework the hierarchy in the CKM mixing angles is accounted for by a corresponding hierarchy in the f_{Q^i} s, $f_{Q^1, Q^2}/f_{Q^3} \sim \lambda_C^3, \lambda_C^2$ with $\lambda_C = 0.23$,

$$\delta_{12} \approx 1, \quad \lambda_{12} \approx \frac{f_{Q^2}^2}{2}. \quad (33)$$

We find the following constraints for the weakest bounds

$$f_{Q^2} \leq \sqrt{\frac{m_{\text{KK}}}{\text{TeV}}} \times \begin{cases} 0.020 & \text{maximal phases} \\ 0.056 & \text{vanishing phases} \end{cases} \quad (34)$$

There are two interesting limits to study. First, electroweak precision measurements put a lower bound of $m_{\text{KK}} \sim 3$ TeV. At the limit, our flavor constraints of Eq. (34) imply $f_{Q^2} \lesssim 0.034$, which translates to $f_{Q^3} \lesssim 0.67$. Second, when the third generation doublet is fully composite, we have $f_{Q^3} = 1$. In this case Eq. (34) gives a lower bound on the KK scale: $m_{\text{KK}} \gtrsim 6.7$ TeV. We conclude that for KK scale at the LHC reach, CPV data exclude the possibility of fully composite third generation doublet (unless the CPV phase is accidentally small, $\sin 2\gamma \lesssim 0.2$). Note that in RS models flavor protection is due to having $\lambda_{12} \ll 1$, while in the supersymmetry case it is due to universality, $\delta_{12} \ll 1$.

Conclusions. The contributions of new physics to FCNC processes depend on four features of the new physics: its scale Λ_{NP} , a possible flavor-blind suppression λ_{12} (loop factors and kinematic functions), flavor degeneracy δ_{12} , and flavor alignment, $\sin \alpha$. Low energy flavor measurements probe new physics only indirectly. Usually, such indirect probes constrain only the product of these four suppression factors, which is represented by the coefficient of a non-renormalizable flavor-changing operator. In this work we show, however, that some combinations of measurements go beyond this simplest set of constraints and probe the new physics in more detail.

In particular, for non-renormalizable operators that involve only SU(2)-doublet quarks, the contributions to $K^0 - \bar{K}^0$ mixing and $D^0 - \bar{D}^0$ mixing depend, to a good approximation, on the same Λ_{NP} , λ_{12} - and δ_{12} -related factors but differ in their alignment factors in a way that depends on the CKM matrix. Thus, the combination of these measurements constrains, for TeV-scale new physics, the product of the flavor-blind and flavor-degeneracy suppression factors [Eqs. (17), (29)]. Within supersymmetry, a strong degeneracy between the first two squark-doublet generations is required [Eq. (31)]. In models of warped extra dimensions, a fully composite third generation quark doublet is generically excluded, unless the KK scale is beyond the LHC reach [Eq. (34)].

Acknowledgments. This research is supported by the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel. The work of YG is supported by the NSF grant PHY-0757868. The work of YN is supported by the Israel Science Foundation founded by the Israel Academy of Sciences and Humanities, the German-Israeli foundation for scientific research and development (GIF), and the Minerva Foundation. The work of GP is supported by the Peter and Patricia Gruber Award.

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